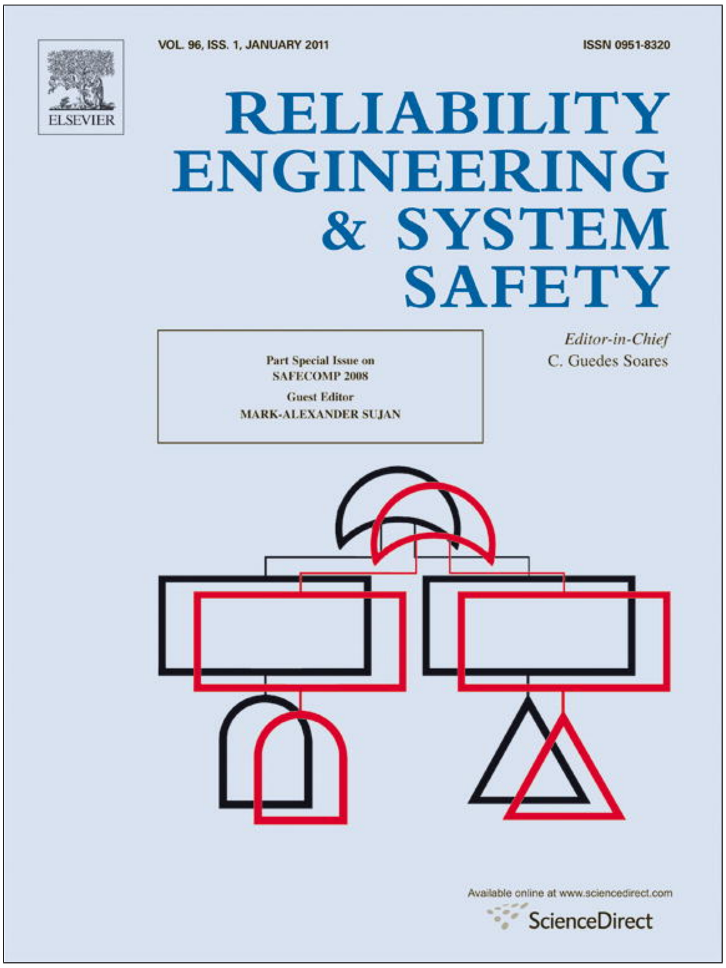
VOL. 96, ISS. 1, JANUARY 2011
ISSN 0951-8320
ELSEVIER
RELIABILITY ENGINEERING & SYSTEM SAFETY
Editor-in-Chief
C. Guedes Soares
Part Special Issue on
SAFECOMP 2008
Guest Editor
MARK-ALEXANDER SUJAN
Available online at www.sciencedirect.com
ScienceDirect

Reliability Engineering and System Safety 96 (2011) 64–74

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

Some considerations on the treatment of uncertainties in risk assessment for practical decision making

Terje Aven [a,*], Enrico Zio [b,c]

[a] University of Stavanger, 4036 Stavanger, Norway
[b] Ecole Centrale Paris- Supelec, Paris, France
[c] Dipartimento di Energia, Politecnico di Milano, Milano, Italy

ARTICLE INFO

Article history:
Received 3 March 2010
Received in revised form
1 June 2010
Accepted 1 June 2010
Available online 8 June 2010

Keywords:
Risk assessment
Uncertainty representations
Decision-making

ABSTRACT

This paper discusses the challenges involved in the representation and treatment of uncertainties in risk assessment, taking the point of view of its use in support to decision making. Two main issues are addressed: (1) how to faithfully represent and express the knowledge available to best support the decision making and (2) how to best inform the decision maker. A general risk-uncertainty framework is presented which provides definitions and interpretations of the key concepts introduced. The framework covers probability theory as well as alternative representations of uncertainty, including interval probability, possibility and evidence theory.

1. Introduction

The aim of this work is to critically reflect on the state of knowledge about the treatment of uncertainties in risk assessments used in practical decision making situations concerning high-consequence technologies, e.g. nuclear, oil and gas, transport etc. The starting point is the acknowledgment that although the use of risk assessment and uncertainty analysis for decision making may take different perspectives, there is a shared and common understanding that these tools provide useful decision support in the sense that their outcomes inform the decision makers insofar as the technical risk side of the problem is relevant for the decision [1].

It is further understood that the actual decision outcome for a critical situation involving a potential for large consequences typically derives from a thorough process which combines (i) an analytic evaluation of the situation (i.e., the risk assessment) by rigorous, replicable methods evaluated under agreed protocols of an expert community and peer-reviewed to verify the assumptions underpinning the analysis, and (ii) a deliberative group exercise in which all involved stakeholders and decision makers collectively consider the decision issues, look into the arguments for their support, scrutinize the outcomes of the technical analysis and introduce all other values (e.g. social and political) not explicitly included in the technical analysis [2]. This way of proceeding allows keeping the technical analysis manageable by complementation with deliberation for ensuring coverage of the non-modeled issues. In this way, the analytic evaluation (i.e., the risk assessment) supports the deliberation by providing numerical outputs (point estimates and distributions of the relevant safety parameters, possibly to be compared with predefined numerical safety criteria for further guidance to the decision) and also all the argumentations behind the analysis itself, including the assumptions, hypotheses, parameters and their uncertainties. With respect to the latter issue, the key point is to guarantee that uncertainties are taken into account in a way that the information and knowledge relevant for the problem are represented in the most faithful manner. The bottom line concern with respect to uncertainty in decision making is to provide the decision makers with a clearly informed picture of the problem upon which they can confidently reason and deliberate.

This process represents an ambition or ideal. The real world is of course often far away from this ideal (refer to discussions for example by Jasanoff [3,4]).

A number of alternative approaches exist for representing and describing uncertainties in risk assessments. Five main categories are:

(a) probabilistic analysis [5],
(b) probability bound analysis, combining probability analysis and interval analysis [6],
(c) imprecise probability, after Walley [70]) and the robust Bayes statistics area [7],
(d) random sets, in the two forms proposed by Dempster [8] and Shafer [9],

* Corresponding author.
E-mail addresses: terje.aven@uis.no (T. Aven), enrico.zio@ecp.fr, enrico.zio@polimi.it (E. Zio).

0951-8320/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ress.2010.06.001

(e) possibility theory [10,11], which is formally a special case of the imprecise probability and random set theories.

For more than 30 years, the probabilistic analysis has been used as the basis for the analytic process of risk assessment (see reviews by Rechard [12,13]). The common term used is Probabilistic Risk Assessment (PRA, also referred to as Quantitative Risk Assessment, QRA). Its first application to large technological systems (specifically nuclear power plants) dates back to the early 1970s [14] but the basic analysis principles have not changed much.

However, the purely probability-based approaches to risk and uncertainty analysis can be challenged under the common conditions of limited or poor knowledge on the high-consequence risk problem, for which the information available does not provide a strong basis for a specific probability assignment: in such a decision making context, many stakeholders may not be satisfied with a probability assessment based on subjective judgments made by a group of analysts. In this view, a broader risk description is sought where all the uncertainties are laid out "plain and flat" with no additional information inserted in the analytic evaluation in the form of assumptions and hypotheses which cannot be proven right or wrong. This concern has sparked a number of investigations in the field of uncertainty representation and analysis, which have led to the developments of frameworks such as those categorized in items (b)–(e) above.

Finally, notice that in the implementation of the decision it is common that the decision makers seek for further protection by adding conservatisms and performing traditional engineering approaches of "defense-in-depth" to bound the uncertainties and in particular the "unknown unknowns" (completeness uncertainty).

In this paper, we will attempt to critically revisit the above mentioned frameworks of uncertainty analysis. The technical details of the different frameworks will be exposed only to the extent necessary to analyze and judge how these contribute to the communication of risk and the representation of the associated uncertainties to decision makers, in the typical settings of high-consequence risk analysis of complex systems with limited knowledge on their behavior. The driver of the critical analysis is really the decision making and the need to feed it with representative information derived from the risk assessment, to robustly support the decision.

The motivation behind the work comes from engineering applications; yet, the discussion is to large extent general and relevant also for other areas, including climate change and financial risk management which dominate the current risk concerns of the World: the role of risk assessments and the way uncertainties are treated in these assessments are of utmost importance for the management and governance of risk in these areas.

2. Risk and risk analysis

The subject of risk nowadays plays a relevant role in the design, development, operation and management of components, systems and structures in many types of industry. In all generality, the problem of risk arises wherever there exist a potential source of damage or loss, i.e. a hazard (threat), to a target, e.g. people or the environment. Under these conditions, safeguards are typically devised to prevent the occurrence of the hazardous conditions, and protections are emplaced to protect from and mitigate its associated undesired consequences. The presence of a hazard does not suffice itself to define a condition of risk; indeed, inherent in the latter there is the uncertainty that the hazard translates from potential to actual damage, bypassing safeguards and protections. In synthesis, the notion of risk involves some kind of loss or damage that might be received by a target and the uncertainty of its transformation in an actual loss or damage.

One classical way to defend a system against the uncertainty of its failure scenarios has been to: (i) identify the group of failure event sequences leading to credible worst-case accident scenarios $\{s^*\}$ (design-basis accidents), (ii) predict their consequences $\{x^*\}$ and (iii) accordingly design proper safety barriers for preventing such scenarios and for protecting from, and mitigating, their associated consequences [15].

Within this approach (often referred to as a *structuralist defense-in-depth approach*), safety margins against these scenarios are enforced through conservative regulation of system design and operation, under the creed that the identified worst-case, credible accidents would envelope all credible accidents for what regards the challenges and stresses posed on the system and its protections. The underlying principle has been that if a system is designed to withstand all the worst-case credible accidents, then it is "by definition" protected against any credible accident [16].

This approach has been the one classically undertaken, and in many technologies it still is, to protect a system from the uncertainty of the unknown failure behaviors of its components, systems and structures, without directly quantifying it, so as to provide reasonable assurance that the system can be operated without undue risk. However, the practice of referring to "worst" cases implies strong elements of subjectivity and arbitrariness in the definition of the accidental events, which may lead to the consideration of scenarios characterized by really catastrophic consequences, although highly unlikely. This may lead to the imposition of unnecessarily stringent regulatory burdens and thus excessive conservatism in the design and operation of the system and its protective barriers, with a penalization of the industry. This is particularly so for those high-consequence industries, such as the nuclear, aerospace and process ones, in which accidents may lead to potentially large consequences.

For this reason, an alternative approach has been pushed forward for the design, regulation and management of the safety of hazardous systems. This approach, initially motivated by the growing use of nuclear energy and by the growing investments in aerospace missions in the 1960s, stands on the principle of looking quantitatively also at the reliability of the accident-preventing and consequence-limiting protection systems that are designed and implemented to intervene in protection against all potential accident scenarios, in principle with no longer any differentiation between credible and incredible, large and small accidents [17]. Initially, a number of studies were performed for investigating the merits of a quantitative approach based on probability for the treatment of the uncertainty associated with the occurrence and evolution of accident scenarios [18]. The findings of these studies motivated the first complete and full-scale probabilistic risk assessment of a nuclear power installation [14]. This extensive work showed that indeed the dominant contributors to risk need not be necessarily the design-basis accidents, a "revolutionary" discovery undermining the fundamental creed underpinning the structuralist, defense-in-depth approach to safety [16].

Following these lines of thought, and after several "battles" for their demonstration and valorisation, the probabilistic approach to risk analysis (PRA) has arisen as an effective way for analyzing system safety, not limited only to the consideration of worst-case accident scenarios but extended to looking at all feasible scenarios and its related consequences, with the probability of occurrence of such scenarios becoming an additional key aspect to be quantified in order to rationally and quantitatively handle uncertainty [14,19–27].

From the view point of safety regulations, this has led to the introduction of new criteria that account for both the consequences of the scenarios and their probabilities of occurrence under a now *rationalist, defense-in-depth approach*. Within this approach to safety analysis and regulation, reliability engineering takes on an important role in the assessment of the probability of occurrence of the accident scenarios as well as the probability of the functioning of the safety barriers implemented to hinder the occurrence of hazardous situations and mitigate their consequences if such situations should occur [15].

2.1. The framework of PRA

The basic analysis principles used in a PRA can be summarized as follows. A PRA systemizes the knowledge and uncertainties about the phenomena studied by addressing three fundamental questions [27]:

- Which sequences of undesirable events transform the hazard into an actual damage?
- What is the probability of each of these sequences?
- What are the consequences of each of these sequences?

This leads to a widely accepted, technical definition of risk in terms of a set of triplets [25] identifying the sequences of undesirable events leading to damage (the accident scenarios), the associated probabilities and the consequences. In this view, the outcome of a risk analysis is a list of scenarios quantified in terms of probabilities and consequences, which collectively represent the risk. On the basis of this information, the designer, the operator, the manager and the regulator can act effectively so as to manage (and possibly reduce) risk.

In the PRA framework, knowledge of the problem and the related uncertainties are systematically manipulated by rigorous and repeatable probability-based methods to provide representative risk outcomes such as the expected number of fatalities (in terms of indices such as potential loss of lives (PLL) and fatal accident rate (FAR)), the probability that a specific person shall be killed due to an accident (individual risk) and frequency-consequence (f–n) curves expressing the expected number of accidents (frequency f) with at least n fatalities.

In spite of the maturity reached by the methodologies used in PRA, a number of new and improved methods have been developed in recent years to better meet the needs of the analysis, in light of the increasing complexity of the systems and to respond to the introduction of new technological systems. Many of the methods introduced allow increased levels of detail and precision in the modeling of phenomena and processes within an integrated framework of analysis covering physical phenomena, human and organisational factors as well as software dynamics (e.g. [28]). Other methods are devoted to the improved representation and analysis of the risk and related uncertainties, in view of the decision making tasks that the outcomes of the analysis are intended to support. Examples of newly introduced methods are Bayesian Belief Networks (BBNs), Binary Digit Diagrams (BDDs), multi-state reliability analysis, Petri nets and advanced Monte Carlo simulation tools. For a summary and discussion of some of these models and techniques, see [23,15].

The probabilistic analysis underpinning PRA stands on two lines of thinking, the traditional frequentist approach and the Bayesian approach [22,23]. The former is typically applied in case of large amount of relevant data; it is founded on well-known principles of statistical inference, the use of probability models, the interpretation of probabilities as relative frequencies, point values, confidence intervals estimation and hypothesis testing.

The Bayesian approach is based on the use of subjective probabilities and is applicable also in case of scarce amount of data. The idea is to first establish adequate probability models representing the aleatory uncertainties, i.e. the variabilities in the phenomena studied, such as for example the lifetimes of a type of unit; then, the epistemic uncertainties (due to incomplete knowledge or lack of knowledge) about the values of the parameters of the models are represented by prior subjective probability distributions; when new data on the phenomena studied become available, Bayes' formula is used to update the representation of the epistemic uncertainties in terms of the posterior distributions. Finally, the predictive distributions of the quantities of interest (the observables, for example the lifetime of new units) are derived by applying the law of total probability. The predictive distributions are subjective but they also reflect the inherent variability represented by the underlying probability models.

From a conceptual viewpoint, a subjective probability is commonly linked to the betting interpretation that goes back to the foundational literature on subjective probabilities (see e.g. [29,30]). However to avoid a mixture between uncertainty assessments and value judgments many analysts prefer to use the comparison with a standard interpretation, for example drawing a ball from an urn [22,31]. The term "subjective probability" is also debated – it gives the impression that the probability and the associated assessment are non-scientific and arbitrary; it is often replaced by terms such as "judgmental probability" and "knowledge-based probability" [30,32,33].

3. Uncertainty and uncertainty analysis

In all generality, the quantitative analyses of the phenomena occurring in many engineering applications are based on mathematical models that are then turned into operative computer codes for simulation. A model provides a representation of a real system dependent on a number of hypotheses and parameters. The model can be deterministic (e.g. Newton's dynamic laws or Darcy's law for groundwater flow) or stochastic (e.g. the Poisson model for describing the occurrence of earthquake events).

In practice, the system under analysis cannot be characterized exactly – the knowledge of the underlying phenomena is incomplete. This leads to uncertainty in both the values of the model parameters and on the hypotheses supporting the model structure. It defines the scope of the *uncertainty analysis*.

An uncertainty analysis aims at determining the uncertainty in analysis results that derives from uncertainty in analysis inputs [34–36]. We may illustrate the ideas of the uncertainty analysis by introducing a model $G(X)$, which depends on the input quantities X and on the function G; the quantity of interest Z is computed by using the model $Z=G(X)$. The uncertainty analysis of Z requires an assessment of the uncertainties of X and their propagation through the model G to produce a characterization of the uncertainties of Z. Typically, the uncertainty related to the model structure G, i.e., uncertainty due to the existence of alternative plausible hypotheses on the phenomena involved, are treated separately [5,37–39]; actually, while the first source of uncertainty has been widely investigated and more or less sophisticated methods have been developed to deal with it, research is still ongoing to obtain effective and accepted methods to handle the uncertainty related to the model structure [40]. See also Aven [1] who distinguishes between model inaccuracies (the differences between Z and $G(X)$), and model uncertainties due to alternative plausible hypotheses on the phenomena involved.

Uncertainty is thus an unavoidable component affecting the behavior of systems and more so with respect to their limits of operation. In spite of how much dedicated effort is put into improving the understanding of systems, components and processes through the collection of representative data, the appropriate characterization, representation, propagation and interpretation of uncertainty remains a fundamental element of the risk analysis of any system. Following this view, uncertainty analysis is considered an integral part of PRA, although it can also exist independently in the evaluation of unknown quantities.

In the context of PRA, uncertainty is conveniently distinguished into two different types: randomness due to inherent variability in the system (i.e. in the population of outcomes of its stochastic process of behavior) and imprecision due to lack of knowledge and information on the system. The former type of uncertainty is often referred to as objective, aleatory or stochastic whereas the latter is often referred to as subjective, epistemic or state-of-knowledge [5,41–44]. Probability models are introduced to represent the aleatory uncertainties, for example a Poisson model to represent the variation in the number of events occurring in a period of time. The epistemic uncertainties arise from a lack of knowledge of the parameters of the probability models. Whereas epistemic uncertainty can be reduced by acquiring knowledge and information on the system, the aleatory uncertainty cannot, and for this reason it is sometimes called irreducible uncertainty.

4. Methods of representation and treatment of uncertainty

The traditional tool used to express the uncertainties in PRA is (subjective) probabilities. In this context, the quantities X and Z referred to in the previous Section could be chances representing fractions in a large (in theory infinite) population of similar items (loosely speaking, a chance is the Bayesian term for a frequentist probability, cf. the representation theorem of de Finetti [45] and Bernardo and Smith [46, p. 172]). In this case, the assessment is consistent with the so-called probability of frequency approach, which is based on the use of subjective probabilities to express epistemic uncertainties of unknown frequencies, i.e. the chances [25]. The probability of frequency approach constitutes the highest level of uncertainty analysis according to a commonly referenced uncertainty treatment classification system [47]. Refer to the Appendix A for further details on the probability of frequency approach; this Appendix A is partly based on Flage et al. [48].

However, the probability-based approaches to risk and uncertainty analysis can be challenged, as discussed in Section 1. Many researchers find the above framework for assessing risk and uncertainties to be too narrow: risk is more than some analysts' subjective probabilities, which may lead to poor predictions of Z. The knowledge that the probabilities are based on could be poor and/or based on wrong assumptions. One may assign a low probability of health problems occurring as a result of some new chemicals, but these probabilities could produce poor predictions of the actual number of people that experience such problems. Or one may assign a probability of fatalities occurring on an offshore installation based on the assumption that the installation structure will withstand a certain accidental load; in real-life the structure could however fail at a lower load level: the assigned probability did not reflect this uncertainty.

Many researchers would argue that the information commonly available in the practice of risk decision making does not provide a sufficiently strong basis for a specific probability assignment; the uncertainties related to the occurrence of the events and associated consequences are too large. Furthermore, in a risk analysis context there are often many stakeholders and they may not be satisfied with a probability-based assessment expressing the subjective judgments of the analysis group: again a broader risk description is sought.

It is true that adopting the subjective probability approach, probabilities can always be assigned, but the information basis supporting the assignments is not reflected by the numbers produced. One may for example assess two situations both resulting in subjective probabilities equal to 0.7 say, but in one case the assignment is supported by substantial amount of relevant data, the other by more or less no data.

Dubois [49] expresses the problem in this way: if the ill-known inputs or parameters to a mathematical model are all represented by single probability distributions, either objective when sufficient information is available or subjective when scarce information is available, then the resulting distribution of the output can hardly be properly interpreted: "the part of the resulting variance due to epistemic uncertainty (that could be reduced) is unclear" [49].

The problem as commented by Dubois [49] seems to be that aleatory uncertainties are mixed with epistemic uncertainties. However, if chances (more generally, probability models with parameters) can be established (justified) reflecting the aleatory uncertainties a full risk description needs to assess uncertainties about these quantities. It would not be sufficient to provide predictive distributions alone, as important aspects of the risk then would not be revealed. The predictive distributions would not distinguish between the stochastic variation and the epistemic uncertainties as noted by Dubois [49]. The indicated inadequacy of the subjective probabilities for reflecting uncertainties is thus more an issue of addressing the right quantities: if chances can be established (justified), the subjective probabilities should be used to reflect the uncertainties about these chances.

Probability models constitute the basis for statistical analysis, and are considered essential for assessing the uncertainties and drawing useful insights [34,50]. The probability models coherently and mechanically facilitate the updating of probabilities. A probability model presumes some sort of model stability, populations of similar units need to be constructed (in the Bayesian context, formally an infinite set of exchangeable random variables). But such stability is often not fulfilled [51]. Consider the definition of a chance. In the case of a die we would establish a probability model expressing that the distribution of outcomes is given by $(p_1, p_2, \ldots, p_6)$, where p_i is the chance of outcome i, interpreted as the fraction of outcomes resulting in outcome i. However, in a risk assessment context the situations are often unique, and the establishment of chances means the construction of fictional populations of non-existing similar situations. Then chances and probability models in general, cannot be easily defined as in the die tossing example; in many cases, they cannot be meaningfully defined at all. For example, it makes no sense to define a chance (frequentist probability) of a terrorist attack [52]. In other cases, the conclusion may not be so obvious. For example, a chance of an explosion scenario in a process plant may be introduced in a risk assessment, although the underlying population of infinite similar situations is somewhat difficult to describe.

There is a huge literature addressing the foundational problems of the probability-based risk assessments. Here are some examples of critical issues raised. Reid [53] argues that there is a common tendency of underestimation of the uncertainties in risk assessments. The disguised subjectivity of risk assessments is potentially dangerous and open to abuse if it is not recognised. According to Stirling [54], using risk assessment when strong knowledge about the probabilities and outcomes does not exist, is irrational, unscientific and potentially misleading. Tickner and Kriebel [55] stress the tendency of decision-makers and agencies not to talk about uncertainties underlying the risk numbers. Acknowledging uncertainty can weaken the authority of the decision-maker and agency, by creating an image of being unknowledgeable. Precise numbers are used as a facade to cover up what are often political decisions. Renn [56] summarizes the critique drawn from the social sciences over many years and concludes that technical risk analyses represent a narrow framework that should not be the single criterion for risk identification, evaluation and management.

Based on the above critiques, it is not surprising that alternative approaches for representing and describing uncertainties in risk assessment have been suggested, such as those of the four categories (b)–(e) listed in the Introduction. The Appendix A at the end of the paper contains minimum details on the concepts underpinning these approaches. For a more in-depth review, see [57].

In probability bound analysis (b), interval analysis is used for those components whose aleatory uncertainties cannot be accurately estimated; for the other components, traditional probabilistic analysis is carried out. However, this results often in very wide intervals and the approach has been criticised for not providing the decision-maker with specific analyst and expert judgments about epistemic uncertainties [33]. The other three frameworks (c)–(e) allow for incorporation and representation of incomplete information. Their motivation is to be able to treat situations where there is more information than an interval, but less than a single specific probability distribution would imply. The theories produce epistemic-based uncertainty descriptions and in particular probability intervals (see the example in the next Section 4.1), but they have not been broadly accepted in the risk assessment community. Much effort has been made in this area, often with a mathematical orientation, but no convincing framework for risk assessment in practice presently exists based on these alternative theories. Further research is required to make these alternatives operational in a risk assessment context.

The same conclusions can be drawn also for other alternative approaches that have recently been suggested, for example the probabilistic inference with uncertain and partial evidence developed by Groen and Mosleh [58] as a generalization of Bayes' Theorem.

Work has also been carried out to combine different approaches, for example probabilistic analysis and possibility theory. Here the uncertainties of some parameters are represented by probability distributions and those of some other parameters by means of possibilistic distributions. An integrated computational framework has been proposed for jointly propagating the probabilistic and possibilistic uncertainties [59]. This framework has previously been tailored to event tree analysis [60] and fault tree analysis [61], allowing for the uncertainties about event probabilities (chances) to be represented and propagated using both probability and possibility distributions. The work has been extended in Flage et al. [62] by comparing the results of the hybrid approach with those obtained by purely probabilistic and possibilistic approaches, using different probability/possibility transformations. See also Helton et al. [57] for some additional examples of such integrative work.

4.1. An illustrative example

As an example of the differences in the outcomes of analyses based on different uncertainty representation and propagation approaches, we report here the results presented in Flage et al. [61] with regards to the fault tree analysis of a stand-by liquid control (SBLC) system of a nuclear boiling water reactor (BWR). The probability (chance) q of occurrence of the top event "SBLC failure on demand" in the fault tree over a fixed mission time $T_M = 31$ days is the outcome of interest. The quantity q depends on the logical structure of the fault tree and the probabilities (chances) of occurrence of component failures, or basic events. The probabilities (chances) $p_i(\lambda_i)$ of occurrence in T_M of the basic events B_i are assumed to be unknown. Here λ_i is a parameter of the underlying failure time distribution of component i. In the analysis, exponential distributions are assumed for the failure times of all components in the system, i.e., $p(\lambda_i) = 1 - exp(-\lambda_i T_M)$.

The failure rates are assumed uncertain and the related information available is such that for some of them a probabilistic representation in terms of lognormal distributions is justified, whereas for others triangular possibility distributions (see Appendix A) are justified. Such uncertainties in the component failure rates values must be propagated through the function that links the basic events probabilities (chances) $p(\lambda_i)$ with the top event probability (chance) q. The probabilistic, the possibilistic and the hybrid representation and propagation approaches are considered.

The cumulative distribution functions of the probability (chance) q for the probabilistic (obtained by 10^6 Monte Carlo samplings), possibilistic (obtained by 10^4 α-cut levels) and hybrid (obtained by 10^5 Monte Carlo samplings and 10^4 α-cut levels) approaches are shown in Fig. 1.

Summary statistics are then typically used to draw conclusions based on these results, possibly by comparison with predefined numerical safety criteria for further guidance to the decisions.

For the probabilistic approach, two quantities of interest are the probability of the top event in terms of the expected value $P(A) = E[q]$, and a percentile value of the uncertainty distribution, say the 95th percentile, Q_{95}. In this case these values equal 0.0179 and 0.0603, respectively.

For the possibilistic approach the core C of the distribution is of relevance, i.e. the interval of values for which possibility equals 1, together with the values of the α-cut interval $D^{(0.05)}$, which represents an interval for which $P(q \in D^{(0.05)}) \geq 0.95$. For our fault tree case we find that C is just a point value equal to {0.0334} and $D^{(0.05)} = [0.00120, 0.0725]$. In the possibilistic approach, all that can be said about the probability that the true value of q lies within the core C is that it is greater than or equal to 0. No further probability structure can be assigned. The interpretation of the

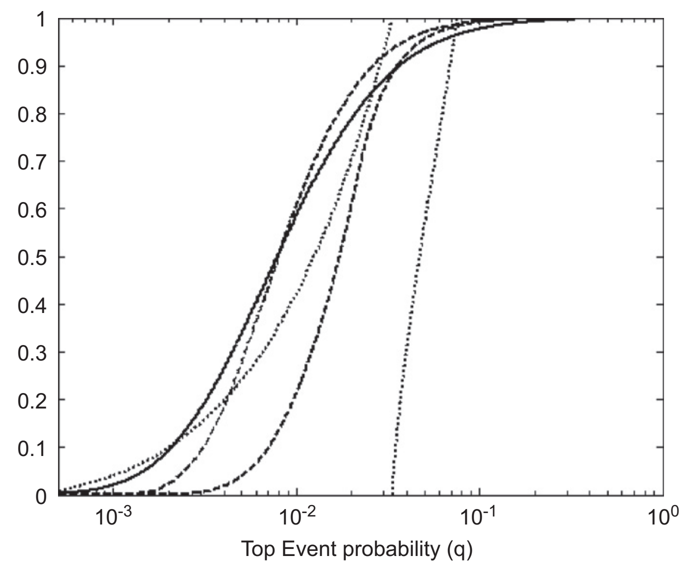

Fig. 1. Cumulative distribution functions for q for probabilistic (solid line), possibilistic (dotted lines) and hybrid (dashed lines) approaches.

α-cut when $\alpha=0.05$, $D^{(0.05)}=[0.00120, 0.0725]$, is thus that there is a 95% or greater probability that q is in $D^{(0.05)}$.

In the hybrid approach, probability and possibility distributions of the failure rates of the basic events are combined to yield lower and upper values of the probability that the top event probability (chance) lies in an interval, say [0, 0.01]. For our fault tree case it turns out that it can only be said that the value of the probability $P(q \leq 10^{-2})$ lies in the interval [0.214, 0.608]. In the risk analysis context, one may then consider the percentile $Q_{95}^{\text{lower}}=0.0476$ which guarantees that the probability that the true value of q is lower than Q_{95}^{lower}, $P(q \leq Q_{95}^{\text{lower}})$, is greater than or equal to 0.95, i.e. $0.95 \leq P(q \leq Q_{95}^{\text{lower}})$.

Hence Q_{95}^{lower} can be interpreted as a conservative assignment of the percentile with respect to the imprecision related to the failure rate probability distributions. If the interval $[0, Q_{95}^{\text{lower}}]$ is considered, the hybrid approach provides also an upper value for $P(q \leq Q_{95}^{\text{lower}})$. In the case study considered this upper bound equals 0.966. The bound [0.95, 0.966] reflects the imprecision about $P(q \leq Q_{95}^{\text{lower}})$ that results from the use of possibility distributions as a representation of uncertainty for some of the failure rates in the fault tree. The analysts (experts) are not able or willing to precisely assign their probability about the values of the failure rates, the result being the above imprecision interval.

All the above mentioned approaches are quantitative. Another research direction is based on a mixture of quantitative and qualitative methods. An example is the semi-quantitative approach outlined by Aven [63–65]. Following this approach, uncertainty factors "hidden" in the background knowledge that the subjective probabilities are based on are identified and assessed in a qualitative way. The motivation for the qualitative analysis is the acknowledgment and belief that the full scope of the risks and uncertainties cannot be transformed to a mathematical formula, using probabilities or other measures of uncertainty. Numbers can be generated but would not alone serve the purpose of the risk assessment, to reveal and describe the risks and uncertainties, as a basis for risk-informed decision-making.

5. Concerns for practical decision making

From the front end of the analysis, the representation of the knowledge available as input to the risk assessment in support of the decision making must be faithful and transparent: the methods and models used should not add information that is not there, nor ignore information that is there. In high-consequence technologies, one deals with rare events and processes for which experimental and field data are lacking or scarce, at best; then it is essential that the related information and knowledge are elicited and treated in an adequate way. Two concerns then need to be balanced:

(i) the knowledge should to the extent possible be "inter-subjective" in the sense that the representation corresponds to documented and approved information and knowledge and
(ii) the risk analysts' judgments (degrees of belief) should be clearly reflected.

The former concern makes the pure Bayesian approach difficult to apply: introducing analysts' subjective probability distributions is unjustifiable since this leads to building a structure in the probabilistic analysis that is not present in the expert-provided information. For example, if an expert states his or her uncertainty assessment of a parameter value in terms of a range of possible values, this does not justify the allocation of a specific distribution function (for example the uniform distribution) onto the range. In this view, it might be said that a more faithful representation of the information and knowledge available would be one that leaves the analysis open to all possible probability distribution structures on the assessed range, without imposing one in particular and without excluding any, thus providing results that bound all possible distributions.

An alternative approach would be to specify a probability distribution based on the maximum entropy principle. This approach does not require the specification of the whole distribution but only of some of its features, for example the mean and variance; then, a mathematical procedure is applied to obtain the distribution characterized by the specified features and, in a certain sense, minimum information beyond that. For further details on the procedure, refer to Bedford and Cooke [23, p. 73].

On the other hand, the representation framework should also take into account the latter concern (ii), i.e. allow for the transparent inclusion of preferential assignments by the experts (analysts) who wish to express that some values are more or less likely than others. The Bayesian approach is the proper framework for such assignments.

From the point of view of the quantitative modeling of uncertainty in risk assessment, two topical issues are the proper handling of dependencies among uncertain parameters, and of model uncertainties. No matter what modeling paradigm is adopted, it is critical that the meaning of the various concepts be clarified. Without such clarifications it is impossible to build a scientific-based risk assessment. In complex situations, when the propagation is based on many parameters, strong assumptions may be required to be able to carry out the analysis. The risk analysts may acknowledge a degree of dependency, but the analysis may not be able to describe it in an adequate way. The derived uncertainty representations must be understood and communicated as measures conditional on this constraint. In practice it is a main task of the analysts to seek for simple representations of the system performance and by smart modeling it is often possible to obtain independence. The models used are also included in the background knowledge of epistemic-based uncertainty representations. We seek accurate models, but at the same time simple models. The choice of the right model cannot be seen in isolation from the purpose of the risk assessments.

From the back-end of the analysis, i.e. the use of its outcomes for practical decision-making, it is fundamental that the meaning and practical interpretation of the quantities computed are communicated in an understandable format to the decision makers. The format must allow for meaningful comparisons with numerical safety criteria if defined, for manipulation (e.g. by screening, bounding and/or sensitivity analyses) and for communication in deliberation processes.

Risk and uncertainty assessments must be seen within the context of decision making. There are in fact also different perspectives on how to *use* risk and uncertainty assessments for decision making. Strict adherence to expected utility theory, cost–benefit analysis and related theories would mean clear recommendations on what is the optimal arrangement or measure. However, most risk researchers and risk analysts would see risk and uncertainty assessments as decision support tools, in the sense that the assessments inform the decision makers. The decision-making is risk-informed, not risk-based [66]. In general, there is a significant leap from the assessments to the decision. What this leap (often referred to as managerial review and judgment) comprises is a subject being discussed in the literature (e.g. [67]) and it is also closely linked to the present work. The scope and boundaries of risk and uncertainty assessments define to a large extent the content of this review and judgment. A narrow probability-based risk and uncertainty characterization calls for a broader managerial review and judgment, and vice versa.

Seeing risk assessment as an aid rather than the sole basis for decision making, alternative approaches for the representation and treatment of uncertainties in risk assessment are required. A Bayesian analysis without thorough considerations of the background knowledge and associated assumptions would normally fail to reveal important uncertainty factors. Such considerations (qualitative assessments) are essential for ensuring that the decision makers are not seriously misled by the risk assessment results.

It is a huge step from the needs and requirements of such assessments to methods that quantitatively express, and bound, the imprecision in the probability assignments. These methods are also based on a set of premises and assumptions, but not to the same degree as the pure probability-based analyses. Their motivation is that the intervals produced correspond better to the information available. In the above fault tree example, the hybrid probability–possibility analysis results in an interval [0.214, 0.608] for the subjective probability $P(q \leq 0.01)$. The risk analysts (experts) are not able or willing to precisely assign their probability $P(q \leq 0.01)$. The decision maker may however request that the analysts make such assignments – the decision maker would like to be informed by the analysts' degree of belief (refer to the concern (ii) above). The analysts are consulted as experts in the field studied and the decision maker expects them to give their faithful report of the epistemic uncertainties about the unknown quantities addressed. The decision-maker knows that these judgments are based on some knowledge and some assumptions, and are subjective in the sense that others could conclude differently, but these judgments are still considered valuable as the analysts have competence in the field being studied. The analysts are trained in probability assignments and the decision-maker expects that the analysts are able to transform their knowledge into probability figures [33].

Following this view, we should continue to conduct probability-based analysis reflecting the analysts' degrees of belief about unknown quantities, but we should also encourage additional assessments. These include sensitivity analyses to see how sensitive the risk indices are with respect to changes in basic input quantities, for example assumptions and suppositions [35,36,68,69], but also crude qualitative assessments of uncertainty factors as mentioned above. The use of imprecision intervals would further point at the importance of key assumptions made.

A unifying framework for representation and treatment of uncertainties in risk assessment can be established following the ideas outlined in the previous Sections. Imprecision intervals constitute an integral part of such a framework. To make these intervals meaningful in a practical decision making context, interpretations such as those provided in Section 4.1 are required. Note that all quantities in Section 4.1 have been interpreted by means of probabilities, because it is common technical language prone to natural interpretation.

The theories for handling imprecision intervals are here not reported in detail (but they are summarized in the Appendix A), as they are technical and not important for the proper understanding of the results. We believe that to make the alternative approaches operational in a practical decision making context, we should leave out the technical terminology used in these theories. If one looks at various attempts that have been made to use alternative representations of uncertainty in risk assessments contexts, the general impression is that they are extremely difficult to understand and appreciate. We believe that in a practical decision making context they would typically be rejected as they add more confusion than insights.

6. Discussion

Nowadays, the use of risk assessment as a tool in support of decision making is quite widespread, particularly in high-consequence technologies. The techniques of analysis sustaining the assessment must be capable of building the level of confidence in the results required for taking the decision they inform. A systematic and rational control on the uncertainty affecting the analysis is the key to confidence building.

In practical risk assessments, the uncertainty is commonly treated by probabilistic methods, in their Bayesian formulation for the treatment of rare events and poorly known processes typical of high-consequence technologies. However, a number of theoretical and practical challenges seem to be still somewhat open. This has sparked the emergence of a number of alternative approaches, which have been here considered in relation to the support to decision making that they can provide.

Many risk researchers and risk analysts are sceptical to the use of "non-probabilistic" approaches (such as those of the four categories (b)–(e) listed in the Introduction) for the representation and treatment of uncertainty in risk assessment for decision making. An imprecise probability result is considered to provide a more complicated representation of uncertainty [31]. By an argument that the simple should be favoured over the complicated, Lindley [31] takes the position that the complication of imprecise probabilities seems unnecessary. In a strong rejection statement, Lindley [71] argues that the use of interval probabilities goes against the idea of simplicity, as well as confuses the concept of measurement (interpretation in the view of Bedford and Cooke [23]) with the practice of measurement (measurement procedures in the view of Bedford and Cooke [23]). The standard for probability assignments that Lindley [71] emphasises (see Appendix A) is a conceptual comparison. It provides a norm, and measurement problems may make the assessor unable to behave according to it. Bernardo and Smith [46] call the idea of a formal incorporation of imprecision into the axiom system "an unnecessary confusion of the *prescriptive* and the *descriptive*" for many applications, and point out that measurement imprecision occurs in any scientific discourse in which measurements are taken. They make a parallel to the inherent limits of a physical measuring instrument, where it may only be possible to conclude that a reading is in the range 3.126–3.135, say. Then, we would typically report the value 3.13 and proceed as if this were the precise number:

We formulate the theory on the prescriptive assumption that we aspire to exact measurement (..), whilst acknowledging that, in practice, we have to make do with the best level of precision currently available (or devote some resources to improving our measuring instruments!) ([46, p. 32]).

Many analysts argue fiercely for a strict Bayesian analysis. A typical statement is [32]: "For me, the introduction of alternatives such as interval analysis to standard probability theory seems a step in the wrong direction, and I am not yet persuaded it is a useful area even for theoretical research. I believe risk analysts will be better off using standard probability theory than trying out alternatives that are harder to understand, and which will not be logically consistent if they are not equivalent to standard probability theory." However, as argued in this paper, this approach does not solve the problems raised. The decision basis cannot be restricted to subjective probabilities: there is a need to go beyond the Bayesian approach.

In the end, any method of uncertainty representation and analysis in risk assessment must address a number of very practical questions before being applicable in support to decision making:

- How completely and faithfully it represents the knowledge and information available?

- How costly is the analysis?
- How much confidence does the decision maker gain from the analysis and the presentation of the results?
- What value does it bring to the dynamics of the deliberation process?

More so, any method which intends to complement, or in some justified cases supplement, the commonly adopted probabilistic approach to risk assessment should demonstrate that the efforts needed for the implementation and familiarization, by the analysts and decision makers, are feasible and acceptable in view of the benefits gained in terms of the above questions and, eventually, of the confidence in the decision made.

7. Conclusions

The present work is seen as a contribution to developing a broad perspective and framework of uncertainty analysis in risk assessment, by critically analyzing alternative approaches and seeking their coherent integration for effective decision making. This perspective and framework extend beyond the Bayesian approach. We argue that a full risk-uncertainty description is more than subjective probabilities. We have outlined and discussed some key issues that we consider important for the development of such a perspective/framework. There are still many unsolved problems and we hope that the paper will stimulate further research in this area, which is considered extremely important for the risk field. If uncertainty cannot be properly treated in risk assessment, the risk assessment tool fails to perform as intended.

Acknowledgments

The inspiration of this work comes from the intense and exciting discussions which took place during a two days workshop in (raining) Los Angeles, at the end of January 2010. For the motivation, the contents and the insights of the paper, the authors are truly indebted to the participants.

The authors are also grateful to the three anonymous reviewers for their useful comments and suggestions to the original version of the paper.

The work of E. Zio has been partially funded by the "Foundation pour une Culture de Securite' Industrielle" of Toulouse, France, under the research contract AO2009-04.

The work of T. Aven has been funded by The Research Council of Norway through the SAMRISK and PETROMAKS research programmes. The financial support is gratefully acknowledged.

Appendix A. Review of methods for representation of uncertainty

A.1. Probability

Probability is a single-valued measure of uncertainty, in the sense that uncertainty about the occurrence of an event A is represented by a single number $P(A)$. Different interpretations of probabilities exist, and these are closely related to different notions of uncertainty. Two interpretations of probability are of widespread use in risk analyses: the relative frequency interpretation and the subjective or Bayesian interpretation.

In the *relative frequency interpretation*, probability is defined as the fraction of times an event A occurs if the situation considered were repeated an infinite number of times. Taking a sample of repetitions of the situation, randomness causes the event A to occur a number of times and to not occur the rest of the times. Asymptotically, this process generates a fraction of successes, the "true" probability $P(A)$. This uncertainty (i.e. variation) is sometimes referred to as aleatory uncertainty. Of course, in practice it is not possible to repeat the experiment an infinite number of times and thus $P(A)$ needs to be estimated, for example by the relative frequency of occurrence of A in the finite sample considered. The lack of knowledge about the true value of $P(A)$ is termed epistemic uncertainty. Whereas epistemic uncertainty can be reduced (by extending the size of the sample), the aleatory uncertainty cannot. For this reason it is sometimes called irreducible uncertainty [73].

In the *subjective (Bayesian) interpretation*, probability is a purely epistemic-based expression of uncertainty as seen by the assigner, based on his/her background knowledge. In this view, the probability of an event A represents the *degree of belief* of the assigner with regard to the occurrence of A. The probability can be assigned with reference to either betting or some standard event. If linked to betting, the probability of the event A, $P(A)$, is the price at which the assessor is neutral between buying and selling a ticket that is worth one unit of payment if the event occurs, and is worthless otherwise [30,45]. Following the reference to a standard, the assessor compares his uncertainty about the occurrence of the event A with some standard events, e.g. drawing a favourable ball from an urn that contains $P(A)\cdot 100\%$ favourable balls [31].

Irrespective of reference, all subjective probabilities are seen as conditioned on the background knowledge K that the assignment is based on. They are probabilities in the light of current knowledge [71]. To show the dependencies on K it is common to write $P(A|\mathrm{K})$, but often K is omitted as the background knowledge is tacitly understood to be a basis for the assignments. Elements of K may be uncertain and seen as unknown quantities, as pointed out by Mosleh and Bier [74]. However, the entire K cannot generally be treated as an unknown quantity and removed using the law of total probability, i.e. by taking $E_{\mathrm{K}}[P(A|\mathrm{K})]$ to obtain an unconditional $P(A)$.

In this view, randomness is not seen as a type of uncertainty in itself. It is seen as a basis for expressing epistemic-based uncertainty. A relative frequency generated by random variation is referred to as a chance, to distinguish it from a probability, which is reserved for expressions of epistemic uncertainty based on belief [30,71]. Thus, we may use probability to describe uncertainty about the unknown value of a chance. As an example, consider an experiment in which the event A of interest occurs $p\cdot 100\%$ of the times the experiment is performed. Suppose that the chance p is unknown. Then, the outcomes of the experiment are not seen as independent, since additional observations would provide more information about the value of p. On the contrary, in the case that p were known the outcomes would be judged as independent, since nothing more could be learned about p from additional observations of the experiment. Thus, conditional on p the outcomes are independent, but unconditionally they are not; they are exchangeable. The probability of an event A for which p is known is simply p. In practice, p is in most cases not known, and the assessor expresses his/her (a priori) uncertainty about the value of p by a probability distribution $H(p)$. Then, the probability of A can be expressed as

$$P(A) = \int P(A|p)\,dH(p) = \int p\,dH(p) \tag{1}$$

One common approach to risk analysis is to use epistemic-based probabilities to describe uncertainty about the true value of a relative frequency-interpreted probability (chance). This is called the *probability of frequency approach* [25] – probability referring to the epistemic-based expressions of uncertainty and frequency to the limiting relative frequencies of events. By taking the expected value of the relative frequency-based probability with respect to the epistemic-based probabilities, both aleatory and epistemic uncertainties are reflected.

A.2. *Imprecise (interval) probability*

To explain the meaning of *imprecise probabilities* (or *interval probabilities*) consider an event A. Then uncertainty is represented by a lower probability $\underline{P}(A)$ and an upper probability $\overline{P}(A)$, giving rise to a probability interval $[\underline{P}(A), \overline{P}(A)]$, where $0 \le \underline{P}(A) \le \overline{P}(A) \le 1$. The difference

$$\Delta P(A) = \overline{P}(A) - \underline{P}(A) \quad (2)$$

is called the imprecision in the representation of the event *A*. Single-valued probabilities are a special case of no imprecision and the lower and upper probabilities coincide. The intervals can be interpreted using de Finetti's gambling framework as shown by Walley [70]: the lower value is the highest price at which the assessor is sure he or she would buy a gamble, and the upper value is the lowest price at which the assessor is sure he or she would be selling the gamble. If the upper and lower values are equal, the interval is reduced to a precise probability.

We may alternatively link the understanding of the interval probability to the reference to a standard interpretation of a subjective probability $P(A)$: the assessor's degree of belief of *A* to occur is stronger than the degree of belief of drawing a favourable ball from an urn which include $\underline{P}(A) \times 100\%$ favourable balls, and weaker than the degree of belief of drawing a favourable ball from an urn which include $\overline{P}(A) \times 100\%$ favourable balls.

We refer to Walley [70] and Coolen and Utkin [75].

A.3. *Probability bound analysis*

Ferson and Ginnzburg [6] suggest a combined probability analysis - interval analysis, referred to as a *probability bound analysis*. The setting is a risk assessment where the aim is to express uncertainties about some parameters θ_i of a model (a function θ of the θ_is, for example θ equal to the product of the parameters θ_i). For the parameters where the aleatory uncertainties cannot be accurately estimated, probability interval analysis is used. In this way uncertainty propagation is carried out in the traditional probabilistic way for some parameters, and intervals are used for others. More specifically it means that:

(1) For parameters θ_i where the aleatory uncertainties cannot be accurately estimated, use interval analysis expressing that $a_i \le \theta_i \le b_i$ for constants a_i and b_i,
(2) For parameters θ_i where the aleatory uncertainties can be accurately assessed, use probabilities (relative frequency-interpreted probabilities) to describe the distribution over θ_i.
(3) Combine 1 and 2 to generate a probability distribution over θ, for the different interval limits. For example, assume that for $i=1$, interval analysis is used with bounds a_1 and b_1, whereas for $i=2$, a probabilistic analysis is used. Then we obtain a probability distribution over $\theta=\theta_1\theta_2$ (say) when $\theta_1=a_1$ and a probability distribution over θ when $\theta_1=b_1$.

Following this approach subjective probabilities are not used. Bounds replace the epistemic-based probabilities.

A.4. *Possibility theory*

In *possibility theory*, uncertainty is represented by using a possibility function $r(x)$. For each *x* in a set *Ω*, $r(x)$ expresses the degree of possibility of *x*. When $r(x)=0$ for some *x*, it means that the outcome *x* is considered an impossible situation. When $r(x)=1$ for some *x*, it means that the outcome *x* is possible, i.e. is just unsurprising, normal, usual [11]. This is a much weaker statement than when probability is 1.

The possibility function *r* gives rise to probability bounds, upper and lower probabilities, referred to the necessity and possibility measures (*Nec*, *Pos*). They are defined as follows.

The *possibility* (*plausibility*) of an event *A*, *Pos*(*A*), is defined by

$$Pos(A) = \sup_{\{x \in A\}} r(x), \quad (3)$$

and the necessity measure $Nec(A)$ is defined by $Nec(A) = 1 - Pos(\text{not } A)$.

Let P(r) be a family of probability distributions such that for all events *A*,

$$Nec(A) \le P(A) \le Pos(A).$$

Then

$$Nec(A) = \inf P(A) \quad \text{and} \quad Pos(A) = \sup P(A) \quad (4)$$

where inf and sup are with respect to all probability measures in P. Hence the necessity measure is interpreted as a lower level for the probability and the possibility measure is interpreted as an upper limit. Using subjective probabilities, the bounds reflect that the analyst is not able or willing to precisely assign his/her probability. He or she can only describe a subset of P which contains his/her probability [10].

A typical example of possibilistic representation is the following [60,76]: we consider an uncertain parameter *x*. Based on its definition we know that the parameter can take values in the range [1,3] and the most likely value is 2. To represent this information a triangular possibility distribution on the interval [1, 3] is used, with maximum value at 2, see Fig. A1.

From the possibility function we define α cut sets $F_\alpha=\{x: r(x) \ge \alpha\}$, for $0 \le \alpha \le 1$. For example $F_{0.5}=[1.5, 2.5]$ is the set of *x* values for which the possibility function is greater than or equal to 0.5. From the triangular possibility distribution in Fig. A1, we can conclude that if A expresses that the parameter lies in the interval [1.5, 2.5], then $0.5 \le P(A) \le 1$.

From (4) we can deduce the associate cumulative necessity/possibility measures $Nec(-\infty, x]$ and $Pos(-\infty, x]$ as shown in Fig. A2. These measures are interpreted as the lower and upper limiting cumulative probability distributions for the uncertain parameter x. Hence the bounds for the interval [1, 2] is $0 \le P(A) \le 1$.

These bounds can be interpreted as for the interval probabilities: the interval bounds are those obtained by the analyst as he/she is not able or willing to precisely assign his/her probability – the interval is the best he/she can do given the information available.

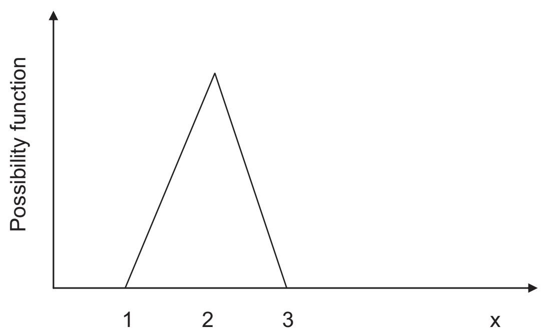

Fig. A1. Possibility function for a parameter on the interval [1, 3], with maximum value at 2.

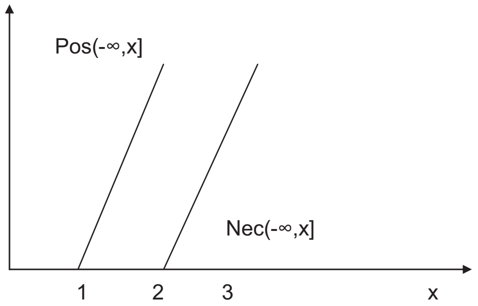

Fig. A2. Bounds for the probability measures based on the possibility function in Fig. A1.

A.5. *Evidence theory (Dempster-Shafer theory. theory of belief functions)*

Random sets in the two forms proposed by Dempster [8] and Shafer [9] is based on the specification of *beliefs* and *plausibilities*, for each subset of outcomes (event) in the sample space under consideration. This allows the theory to take into account the weight of evidence. Possibility theory can be considered a special case of this theory.

Consider an event A and its complement A^c. These are mutually exclusive and exhaustive events, and in probability theory their respective probabilities are required to sum to one. Thus, if the event A is assigned the probability p, then A^c must be assigned the probability $1-p$. To the contrary, in evidence theory degrees of belief are assigned based on the strength of the supporting evidence: the belief value must represent the degree to which evidence is judged to support a given proposition and the degree of belief is explained by Shafer [9] as the commitment of a certain portion of someone's belief. If there is little evidence both in favour of and against the event A, then the belief in both its occurrence and its non-occurrence should be assigned low values. In the extreme case of no evidence at all, both beliefs should be set equal to zero. Letting $Bel(A)$ denote the degree of belief that A will occur and $Bel(A^c)$ the degree of belief that A will not occur, the requirement is only that

$$Bel(A)+Bel(A^c)\leq 1 \tag{5}$$

Thus, the specification of the belief function is capable of incorporating a lack of confidence in the occurrence of the event A, quantitatively manifested in the sum of the beliefs of the occurrence ($Bel(A)$) and non-occurrence ($Bel(A^c)$) being less than one. The difference $1-[Bel(A)+Bel(A^c)]$ is called *ignorance*. When the ignorance is 0, the available evidence justifies a probabilistic description of the uncertainty.

According to Shafer [9], an adequate summary of the impact of evidence must include at least two items of information: the support of the evidence in favour and the support of the evidence against. The plausibility of the event A, $Pl(A)$, is then introduced as the extent to which evidence does not support A^c and the relation between plausibility and belief is

$$Pl(A)=1-Bel(A^c) \tag{6}$$

A fundamental property of the plausibility function is that:

$$Pl(A)+Pl(A^c)\geq 1 \tag{7}$$

Thus, the specification of the plausibility function reflects the evidence in support of the occurrence or not of the event A, as quantified by the sum of the plausibilities of the occurrence ($Pl(A)$) and non-occurrence ($Pl(A^c)$) being greater than or equal to one.

The theory is based on the idea of obtaining degrees of belief for one question from subjective probabilities for related questions [72]. To illustrate, suppose that a diagnostic model is available to indicate with reliability (i.e. probability of providing the correct result) of 0.9 when a given system is failed. Considering a case in which the model does indeed indicate that the system is failed, this fact justifies a 0.9 degree of belief on such event (which is different from the related event of model correctness for which the probability value of 0.9 is available) but only a 0 degree of belief (not a 0.1) on the event that the system is not failed. This latter belief does not mean that it is certain that the system has failed, as a zero probability would: it merely means that the model indication provides no evidence to support the fact that the system is not failed. The pair of values {0.9, 0} constitutes a belief function on the propositions "the system is failed" and "the system is not failed".

From the above simple example, one can appreciate how the degrees of belief for one question (has the system failed?) are obtained from probabilities related to another question (is the diagnostic model correct?).

Denoting by A the event that the system is failed and by m the diagnostic indication of the system state, the conditional probability $P(m|A)$, i.e. the model reliability, is used as the degree of belief that the system is failed. This is unlike the standard Bayesian analysis, where focus would be on the conditional probability of the failure event given the state diagnosis by the model, $P(A|m)$, which is obtained by updating the prior probability on A, $P(A)$, using Bayes' rule.

As for the interpretation of the measures introduced in evidence theory, Shafer [72] uses several metaphors for assigning (and hence interpreting) belief functions. The simplest says that the assessor judges that the strength of the evidence indicating that the event A is true, $Bel(A)$, is comparable with the strength of the evidence provided by a witness who has a $Bel(A)\cdot 100\%$ chance of being reliable. Thus, we have

$$Bel(A)=P(\text{The witness claiming that A is true is reliable}) \tag{8}$$

The metaphor is to be interpreted as the diagnostic model analyzed above, witness reliability playing the role of model reliability.

References

[1] Aven T. Some reflections on uncertainty analysis and management. Reliability Engineering and System Safety 2010;95:195–201.
[2] National Research Council. Understanding risk: informing decisions in a democratic society. Washington, DC: National Academy Press; 1996.
[3] Jasanoff S. Risk management and political culture. N.Y: Russell Sage Foundation; 1986.
[4] Jasanoff S. Science at the bar. Harvard University Press; 1995.
[5] Apostolakis G. The concept of probability in safety assessments of technological systems. Science 1990;250:1359–64.
[6] Ferson S, Ginnzburg. Different methods are needed to propage ignorance and variability. Reliability Engineering and System Safety 1996;54:133–44.
[7] Berger J. An overview of robust Bayesian analysis. Test 1994;3:5–124.
[8] Dempster A. Upper and lower probabilities induced by a multivalued mapping. Annals of Mathematical Statistics 1967;38:325–39.
[9] Shafer G. A mathematical theory of evidence. Princeton: Princeton University Press; 1976.
[10] Dubois D, Prade H. Possibility theory. New York: Plenum Press; 1988.
[11] Dubois D. Possibility theory and statistical reasoning. Computational Statistics & Data Analysis 2006;51:47–69.
[12] Rechard RP. Historical relationship between performance assessment for radioactive waste disposal and other types of risk assessment. Risk Analysis 1999;19(5):763–807.
[13] Rechard RP. Historical background on performance assessment for the waste isolation pilot plant. Reliability Engineering and System Safety 2000;69(3): 5–46.
[14] NRC. Reactor safety study, an assessment of accident risks. Wash 1400. Report NUREG-75/014. U.S. Nuclear Regulatory Commission, Washington, DC, 1975.
[15] Zio E. Reliability engineering: old problems and new challenges. Reliability Engineering and System Safety 2009;94:125–41.
[16] Apostolakis GE. PRA/QRA: an historical perspective. In: 2006 Probabilistic/ quantitative risk assessment workshop, Taiwan, 29–30 November 2006.

[17] Farmer FR. The growth of reactor safety criteria in the United Kingdom. In: Anglo-Spanish power symposium, Madrid, 1964.
[18] Garrick BJ, Gekler WC. Reliability analysis of nuclear power plant protective systems, 1967. USAtomic Energy Commission; 1967. HN-190.
[19] NRC. Severe accident risks: an assessment for five U.S. nuclear power plants. NUREG-1150, vols. 1–3. Washington, DC: U.S. Nuclear Regulatory Commission, Office of Nuclear Regulatory Research, Division of Systems Research; 1990–1991.
[20] Breeding RJ, Helton JC, Gorham ED, Harper FT. Summary description of the methods used in the probabilistic risk assessments for NUREG-1150. Nuclear Engineering and Design 1992;135(1):1–27.
[21] NASA. Probabilistic risk assessment procedures guide for NASA managers and practitioners. NASA; 2002.
[22] Aven T. Foundations of risk analysis. NJ: Wiley; 2003.
[23] Bedford T, Cooke R. Probabilistic risk analysis. Foundations and methods. Cambridge: Cambridge University Publishing Ltd.; 2001.
[24] Henley EJ, Kumamoto H. Probabilistic risk assessment. New York: IEEE Press; 1992.
[25] Kaplan S, Garrick BJ. On the quantitative definition of risk. Risk Analysis 1981;1:11–27.
[26] McCormick NJ. Reliability and risk analysis. New York: Academic Press; 1981.
[27] NUREG. PRA procedures guide. NUREG/CR-2300, vols. 1&2, January 1983.
[28] Mohaghegh Z, Kazemi R, Mosleh A. Incorporating organizational factors into probabilistic risk assessment (PRA) of complex socio-technical systems: a hybrid technique formalization. Reliability Engineering and System Safety 2009;94(2009):1000–18.
[29] de Finetti B. Fondamenti logici del ragionamento probabilistico. Bollettino-dell'Unione Matematica Italiana 1930;9:258–61.
[30] Singpurwalla N. Reliability and risk. A Bayesian perspective. NJ: Wiley; 2006.
[31] Lindley DV. The philosophy of statistics. The Statistician 2000;49(3): 293–337.
[32] North W. Comments on Aven's paper. Risk Analysis 2010;30:377–80.
[33] Aven T. On the need for restricting the probabilistic analysis in risk assessments to variability. Risk Analysis. With Discussion 2010;30(3): 354–60 and 381–4.
[34] Helton JC. Treatment of uncertainty in performance assessments for complex systems. Risk Analysis 1994;14:483–511.
[35] Helton JC, Johnson JD, Sallaberry CJ, Storlie CB. Survey of sampling-based methods for uncertainty and sensitivity analysis. Reliability Engineering & System Safety 2006;91:1175–209.
[36] Cacuci DG, Ionescu-Bujor MA. Comparative review of sensitivity and uncertainty analysis of large-scale systems—II: Statistical methods. Nuclear Science and Engineering 2004;147(3):204–17.
[37] Nilsen T, Aven T. Models and model uncertainty in the context of risk analysis. Reliability Engineering & Systems Safety 2003;79:309–17.
[38] Devooght J. Model uncertainty and model inaccuracy. Reliab Engng Syst Safety 1998;59:171–85.
[39] Zio E, Apostolakis GE. Two methods for the structured assessment of model uncertainty by experts in performance assessments of radioactive waste repositories. Reliab Engng Syst Safety 1996;54:225–41.
[40] Parry, G, Drouin, MT. Risk-informed regulatory decision-making at the U.S. NRC: dealing with model uncertainty. Nuclear Regulatory Commission, 2009.
[41] Parry G, Winter PW. Characterization and evaluation of uncertainty in probabilistic risk analysis. Nuclear Safety 1981;22(1):28–42.
[42] Hoffman FO, Hammonds JS. Propagation of uncertainty in risk assessments: the need to distinguish between uncertainty due to lack of knowledge and uncertainty due to variability. Risk Analysis 1994;14(5):707–12.
[43] Helton JC, Oberkampf WL, editors. Special issue reliab eng syst safety, 85; 2004. p. 1–369.
[44] Helton JC, Oberkampf WL. Guest editorial: alternative representation of uncertainty. Reliability Engineering and System Safety 2004;85:1–10.
[45] de Finetti B. Theory of probability. New York: Wiley; 1974.
[46] Bernardo JM, Smith AFM. Bayesian theory. Chichester: Wiley; 1994.
[47] Paté-Cornell ME. Uncertainties in risk analysis: six levels of treatment. Reliability Engineering and System Safety 1996;54(2–3):95–111.
[48] Flage R, Aven T, Zio E. Alternative representations of uncertainty in reliability and risk analysis – review and discussion. In: Martorell S, Guedes Soares C, Barnett J, editors. Valencia, Spain, 22–25 September 2008 Safety, reliability and risk analysis. Theory, methods and applications. Proceedings of the European safety and reliability conference 2009 (ESREL 2009). London: CRC Press; 2009. p. 2081–91.
[49] Dubois D. Representation, propagation and decision issues in risk analysis under incomplete probabilistic information. Risk Analysis 2010;30:361–8.
[50] Winkler RL. Uncertainty in probabilistic risk assessment. Reliability Engineering and System Safety 1996;85:127–32.
[51] Bergman B. Conceptualistic pragmatism: a framework for Bayesian analysis? IIE Transactions 2009;41:86–93.
[52] Aven T, Heide B. Reliability and validity of risk analysis. Reliability Engineering and System Safety 2009;94:1862–8.
[53] Reid SG. Acceptable risk. In: Blockley DI, editor. Engineering safety. New York: McGraw-Hill; 1992. p. 138–66.
[54] Stirling A. Science, precaution and risk assessment: towards more measured and constructive policy debate. European Molecular Biology Organisation Reports 2007;8:309–15.
[55] Tickner J, Kriebel D. The role of science and precaution in environmental and public health policy. In: Fisher E, Jones J, von Schomberg R, editors. Implementing the precautionary principle. Northampton, MA, USA: Edward Elgar Publishing; 2006.
[56] Renn O. Three decades of risk research: accomplishments and new challenges. Journal of Risk Research 1998;1(1):49–71.
[57] Helton JC, Johnson JD, Oberkampf WL. An exploration of alternative approaches to the representation of uncertainty in model predictions. Reliability Engineering and System Safety 2004;85(1–3):39–71.
[58] Groen FJ, Mosleh A. Foundations of probabilistic inference with uncertain evidence. International Journal of Approximate Reasoning 2005;39:49–83.
[59] Baudrit C, Dubois D, Guyonnet D. Joint propagation of probabilistic and possibilistic information in risk assessment. IEEE Transactions on Fuzzy Systems 2006;14:593–608.
[60] Baraldi P, Zio E. A combined Monte Carlo and possibilistic approach to uncertainty propagation in event tree analysis. Risk Analysis 2008;28(5): 1309–25.
[61] Flage R, Baraldi P, Ameruso F, Zio E, Aven T. Handling epistemic uncertainties in fault tree analysis by probabilistic and possibilistic approaches. In: Bris R, Guedes Soares C, Martorell S, editors. Prague, Czech Republic, 7–10 September 2009 Reliability, risk and safety: theory and applications. supplement proceedings of the European safety and reliability conference 2009 (ESREL 2009). London: CRC Press; 2010. p. 1761–8.
[62] Flage R, Baraldi P, Zio E, Aven T. Probabilistic and possibilistic treatment of epistemic uncertainties in fault tree analysis. Submitted for publication.
[63] Aven T. Risk analysis. NJ: Wiley; 2008.
[64] Aven T. A semi-quantitative approach to risk analysis, as an alternative to QRAs. Reliability Engineering and System Safety. 2008;93:768–75.
[65] Aven T. On how to define, understand and describe risk. Reliability Engineering and System Safety. 2010;95(6):623–31.
[66] Apostolakis GE. How useful is quantitative risk assessment?Risk Analysis 2004;24:515–20.
[67] Aven T. Misconceptions of risk. Chichester: Wiley; 2010.
[68] Saltelli A, Ratto M, Andres T, Campolongo F, Cariboni J, Gatelli D, Saisana M, Tarantolla S. Global sensitivity analysis: the primer. New York: Wiley; 2008.
[69] Frey HC, Patil SR. Identification and review of sensitivity analysis methods. Risk Analysis 2002;22(3):553–78.
[70] Walley P. Statistical reasoning with imprecise probabilities. New York: Chapman and Hall; 1991.
[71] Lindley DV. Understanding uncertainty. Hoboken, NJ: Wiley; 2006.
[72] Shafer G. Perspectives on the theory and practice of belief functions. International Journal of Approximate Reasoning 1990;4:323–62.
[73] Helton JC, Burmaster DE. Specuial issue on treatment of aleatory and epistemic uncertainty. Reliability Engineering and System Safety 1996; 54(2–3).
[74] Mosleh A, Bier VM. Uncertaity about probability: a reconciliation with the subjectivist viewpoint. IEEE Trans. on Systems. Man and Cyber. Part A: Systems ansd Humans 1996;26:303–10.
[75] Coolen FPA, Utkin LV. Imprecise probability: a concise overview. In: Aven T, Vinnem JE. edtors. Risk, reliability and societal safety: proceedings of the European safety and reliability conference (ESREL), Stavanger, Norway, 25–27 June 2007. London: Taylor & Francis, 1959–66.
[76] Anoop MB, Rao KB. Determination of bounds on failure probability in the presence of hybrid uncertainties. Sadhana 2008;33:753–65.